Consider the 2-D velocity field $\vec{V}=u\hat{\imath}+v\hat{\jmath}$

u = y v = x

whose hyperbolic-shape streamlines are sketched below. The density ρ is everywhere constant.

a) Evaluate the mass-flow integral

$$I \;=\; \oint \rho \, (\vec{V} \cdot \hat{n}) \; dA$$

for the square control volume of side length ℓ shown in the figure.

Hint: First express I as four separate integrals I_1 , I_2 , I_3 , I_4 over each of the four segments of the C.V. boundary. Note that dA is a length in 2-D.

b) Does this flow satisfy the mass conservation law *everywhere*? Explain.

